Mathability and Coaching

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***Abstract*—In this paper, we point out some benefits of coaching in education in relation with mathability.**

***Keywords*—Mathability; Coaching; Computer Aided Education; Cognitive Abilities.**

INTRODUCTION

Contemporary tools of cognitive infocommunication and easy access to any required information have been changing the way of human cognition and knowledge assimilation. Modern mathematical and technical education should fit the new habits. It is realisable thanks to high mathability level devices and applications. In the paper [**?**], we stated that uncontrolled computer assisted self-education can be risky since a lack of accuracy and sketchy solutions are characteristic for the young generation. We prooved that abilities of selection, assessment of gathered information and reflection on the obtained result are necessery elements of self-education and education in general. We gave examples of computer assisted mentoring in discovering and applying mathematics.

Coaching, specially coaching in education is a method which fits to that way of possessing knowledge. The steps of the procedure described in the presented paper correspond to the well known Bloom's taxonomy of educational objectives (cf. [**?**]). It was shown in [**?**] that the taxonomy levels are suitable to describe mathability levels of machines, as well. Some corresponding examples were presented also (among others) in [**?**], [**?**], [**?**], [**?**], [**?**], [**?**] and [**?**]).

Combination of the three aspects: mathabbility of devices, education and coaching, can form a perfect tool of a modern personal development in relation not only to science but to any discipline human need to develop.

In the present paper we would like to examine usefulness of chosen coaching tools on the base of

- computer assisted ways of assimilating mathematical knowledge or possessing mathematical abilities,
- approaches to application of learnt patterns.

Firstly, a phenomenon of coaching is described as a personal development tool. Its application to education is exemplified. In the second part, examples of application of coaching methods in computer assisted teaching are presented. A new instructive proposal of education and self-education with high-level mathability devices is given in part 3.

I. COACHING

In 1979 Burton and Brown characterized coaching as "guided discovery learning" in which students construct their new knowledge from their existing knowledge, what was an implication of Piaget's theory. What is interesting, the notion was described in *An investigation of computer coaching for informal learning activities* (see [**?**]).

The main coaching rule is a coach neither provides answers nor gives advices. The core goal of method is to support development and the only tool is asking questions to bring out dormant humen potential and boost their capability. This is why one can regard Socrates a pioneer of coaching.

I.1. Definition and coaching basics

Let us invoke a simple definition given by J. Rogers ([**?**]):

coach works with clients in other to significantly and firmly improve their efectivenes in personal and professional life. The only coach'es goal is the clients reveal and develop their potential as they define it on their own.

This point of view leads to formulate six basic rules of coaching distinguishing it from other disciplines (mentoring, therapy, etc.):

1. the client is a root of resources;
2. coach asking questions makes the client begin to use the resources;
3. coaching refers to the present and future of the client;
4. the subject is chosen by the client ;
5. the coach and the client are on par;
6. the goals of coaching are change or operation.

Regardless of a coaching type (e.g. sport, life, business, executive) the above rules are principles which have to be kept strictly. For more details see also [**?**].

One of the first model of a coaching meeting is **GROW**. The name describes a framework of the session and order of questions asked. **G** stands for **goal** since formulating goals is the first step in the method. Next, it is necessary to assess the present and answer the question how it is now in relation

to the goal the coachee wants to achieve. Hence, **R** stands for **reality**. **O** is for checking available **options**. The coachee should generate as many ideas as possible. At the end a description of planned activities should appear, hence **W** is for the coachee **will**.

Although several different models were introduced, GROW is one of the most popular and most frequently used.

It should be noticed that to be successful usually six coaching sessions should be planned and usually before each of them a coachee has to work on some tasks.

For further investigation we will use advices of an experienced coach, Michael Bungay Stanier. In his book *The Coaching Habit: Say Less, Ask More & Change the Way You Lead Forever* (cf. [**?**]) he presents questions and tools thanks to which leaders can easily achieve their goals working less than usually. The author presents seven **golden questions** useful in most cases:

1. Introductive: **What are you thinking about?**
2. Strong: **What else?**
3. Focusing concentration: **In this case, what represents a real challenge for you?**
4. Base: **What do you need? What do you want to?**
5. Lazy: **How can I help?**
6. Strategic: **What should you give up if you make your decision?**
7. Educative: **What was the most useful for you?**

I.2. Coaching in education

According to [**?**], coaching supports an idea of learning to be personalized and challenging. It is a method which provides tools for building up self-determination and eliminating educational stress. The tool is needed by teachers since lack of determination and bullying which accompany lessons of mathematics and other sciences, are one of the main obstacles for enjoying to learn the subjects. Coaching is a mean to harness the energy and positive thinking for education of sciences. It supports, encourages and challenges both students and educators.

In our paper we will follow [**?**] and use the following definition:

coaching in education *is "one-to-one conversation focused on the enhancement of learning and development through increasing self-awarness and a sense of a personal responsibility, where the coach facilitates the self-directed learning of the coachee through guestioning, active listening and appropriate challenge, in a supportive and encouraging climate"*.

If so, all the stages of GROW model as well as appropriately formulated golden questions mentioned in the previous part, are suitable to be used in education.

Creasy and Paterson (cf. [**?**]) defined five key skills aligned with the defined **non-directive coaching** in education:

- establishing rapport and trust,
- listening for meaning,
- questioning for understanding,
- prompting actions, reflection and learning,
- developing confidence and celebrating success.

Briefly it can be said that transfer coaching into education allows educators to use solution-centered coaching technique. The coach edits the frame from targeted questions, confirming feedbacks and useful summaries. The coachee receives space and time to arrange thoughts, to specify some goals, to make real dormant resources and to plan the next step [**?**].

Another sort of the method called *instructive coaching* is addressed to educators and provides procedures of implementation specific teaching methods (cf. [**?**]). Instructional coaches are collaborative, prepare teaching materials, model best practices, provide feedback, etc. However, we will skip this method and focus on the edu-coaching as a teaching technique. For those who got inspired we refer to [**?**].

II. Coaching Methods in Computer Assisted Teaching

High mathability level devices enable to exemplify abstract mathematical notions. On the other hand applying IT allows students to collect information from various sources and yields developing creative thinking. It should encourage students to well-considered actions, since it is necessary to plan which system or application and in which way it should be used in order to obtain the required solution (cf. [**?**]). In this part, we will focus on such a controlled usage of high mathability level applications and explain how useful is coaching in mathability related education.

Example 1.

Helping students to build up new knowledge and skills

In the paper [**?**] we observed how students of a lower secondary school (at the age of 14) associate new knowledge to the prior knowledge system. The students' aim was to present numbers 47 and 126 using the Fibonacci coding. Pupils had already learnt binary coding and, just before the task, they had been presented Fibonacci sequence. Let us recall a Wikipedia brief explanation found by students in a couple of steps. Firstly, they read the definition:

"Fibonacci coding is a universal positional code which encodes positive integers into binary code words. It is one example of representations of integers based on Fibonacci numbers."

Further reading gives the next important information:

- no code word can contain two consecutive 1, i.e. the "11" is not allowed,
- the Fibonacci system is defined as: 0, 1, 1, 2, 3, 5, 8, $\ldots$ and the two first number are never taken into consideration.

Such a method allows to present any positive natural number in a unique form.

Although pupils presented variety of results none of them were correct. They read the description not carefully enough omitting the rule that doubled "one" is not allowed. Hence, they did not ensure the uniqueness of the representation. They had found main key words: "Fibonacci" and "binary coding", however they applied it incorrectly. Even if some of the students had read the definition carefully, they did not follow any solved examples.

Independently of the way of solution they had chosen, they usually made mistakes since they did not pay enough attention to details. We concluded that the ability of reading with understanding and selecting proper content have the same importance. However, selection of information is the weakest point of the process.

Now, we can repeat the exercise applying **the coaching technique**. First of all, we assume that after regular IT classes students have proper knowledge on binary coding, they understand the need of a unique number representation. Moreover, they had already learned Fibonacci numbers and had discovered basic notion of Fibonacci coding. It means that the prior knowledge is **the students inner root of resources**. We will follow the method of **seven golden questions** which can be asked by the educator to help students to select and assess information. We will adapt the questions to our needs.

For instance, after the problem is presented, the teacher can ask:

- What is your first idea?
- WHAT ELSE?
- What is a real problem to you?
- What would you like to achieve? Do you find the result correct? Why?
- How can I help you? Did you used all clues?
- What was the most useful for you? Make it stick!

Of course, it is nothing new, nothing more than a good educator should do while teaching logical thinking or using a problem solving method. Although we know the rules from our study in education, we teach children to think logically rather rarely. Some easy questions asked often enough may help educators to train that ability every day, during each lesson. The great adventage of edu-coaching is it breaks students incapacity to tell about what they do not know and to realize why they are not able to find a solution althoug they have all necessary data. This is the most difficult element of any problem solving method: to name the problem.

Example 2.

Computer assisted discovery

Pupils of a high school, from a class with mathematical profile, discussed chosen problems of mathematical analysis outreaching school material. Computer assisted methods used to gain new knowledge enabled students to obtain surprising results in understanding the notion of convergence of a series (for more details see [**?**]).

After introducing the notion of a limit of a sequence the young people were asked to calculate an infinite sum of the form:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots=?$$

We can easily use **solution-centered technique of golden questions** to lead students to their discovery. We can observe that formerly students were acquainted with the definition of a limit of a sequence, the method of computing any finite partial sum of the given form using appropriate computer application, i.e. they possessed **inner resources** of the knowledge.

The questions to be asked by a teacher (T) and exemplary answers of a student (S) could be as follow:

T: What is your aim?
S: I want to sum up numbers.

T: What are you able to compute?
S: Any finite sum of this form.

T: What is a real probem? What distinguishes this sum from the ones you can compute?
S: The sum is infinite.

T: Do you have any suggestions?
S: I can observe progressively longer sums.

T: What tools do you need to examine it?
S: I will use computer programming or a computer algebra system.

T: What is your observation?
S: After several iterations adding the next fraction almost does not change the result.

T: What is the result?
S: I gues that 2.

T: What makes you think so?
S: After a finite number of steps the results for any finite sum do not differ significantly.

T: You are right. Congratulation! You have just defined the convergence of a series. Tell me, what was the most useful hint for you?
S: Easy calculation of arbitrarily long sum, observing graphically how the partial results behave and comparison to the definition of a sequence limit.

In fact, Wolfram Mathematica was applied to compute consecutive partial sums of the given form. Thanks to that pupils could easily interpret partial sums on the graph and understand their meaning. Moreover, pupils were able to formulate convergence criterion. They realized on their own that the infinite sum would exist if the sequence of partial sums was convergent. Pupils noticed that the infinite sum would be exactly a finite limit of the partial sum sequence.

Undoubtedly, applying computer aided methods and appropriate planned questioning helps pupils to interpret partial results, to boost their creative and logical thinking on their way to new discoveries.

III. Coaching and Mathability General Proposal of the Method

In this section we will consider mathability broadly as human and machine ability to solve mathematical problems.

We will assume that:

- the student has an adequate prior knowledge or
- the student can collect required information and is able to apply a proper mathability level device or application to process the data.

This way we can insure **the student is a root of the resources**. As we exemplified in part II, **solution focused, non-directive coaching** can lead students to the required result by **asking well planned questions**. It is important to create such a situation that **the educator and the student are on par**. The student should feel free and want to cooperate, should not be ashamed. This way we fulfill all six main rules of coaching, mentioned in part I.1.

Proposal 1.
Transactive coaching in relation to mathability

The examples presented in part II as well as the Proposal 1. are related to the idea of **transactive coaching**, i.e the technique **focused on solutions**.

Now we will use GROW model to plan a single session (usually five to six session build a process):

G: The student formulates the **goal** of the session.

R: The student **review** prior knowledge or information formerly found or derived with mathability devices.

O: The student generates as many ideas as possible, checks available **options**.

W: A description of planned activities is presented. Students shows his/her **will** to solve problem.

Of course, all the stages should be accomplished with appropriate questioning, as exemplified in part II.

Proposal 2.
Transformative coaching in relation to mathability

Many cited references focus on the transactive coaching, pointing out how to develop, for instance, ability of logical thinking and solving problems. Nevertheless, we can also help those students who are usually not able to get closer to any solution. **Transformative coaching**, the technique **focused on a positive change**, can help us with the purpose. Again, using questions typical for a regular coaching session we can change students' attitude to mathematics, sciences or problem solving in general.

The questions, first of all, should help the educator to cope with **limiting beliefs** such as: "mathematics is difficult", "if you are humanist then it is clear you are not good at mathematics", "I am not able to learn programming" or more general: "I am simply to weak to do it", "I will certainly fail", etc. When the asked questions are provoking enough then not knowing seems to be a challenge not a shame.

Also in this case we can effectively use GROW model. Here there are some proposed exemplary questions.

G: The student formulates the **goal** of the session answering questions:

- What would you like to achieve?
- What are you thinking about?

R: The student overview the **reality**.

- Up till now, what has been your greatest success considering your goal?
- What has been your failure? How have you felt then?

O: The student generates ideas, checks available **options**.

- **Imagine you are right and successful**.
- How do you feel then?
- Who is proud of you then? When it can happen?

W: The student present a **will** of changes.

- What disturbs you to feel that way?
- What are the reasons you are afraid of consistent testing your ideas?

Summarising

Coaching is a fenomenal method and a perfect tool of changing one's attitude toward their own problem solving. It brings out human's dormant potential using only one's own root of resources. Applying the language of education, we can say that coaching modifies students attitude toward mathematical, phisical, engineering problem solving using the students resources of knowledge gathered formerly, often with IT tool. The task of a coach is to ask appropriate questions in order to encourage a coachee to realize how rich and useful is his/her knowledge and find out a way of applying the knowledge to build a solution consistently. Such an approach to the modern mathematical and technical education would fit the habits of a young generation since the coach can help the coachee to pay more attention to selection and assessment of information and, at the end of the process, to provide reflection on the obtained result.

Thanks to contemporary tools of cognitive infocommunication, coaching in computer assisted education appears a valuable method of discovering and applying mathematics, technics and other sciences.

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